

## 04 Integers

### **GUESS THE BIRTHDAY DATE**

The table you can see below is the base of the activity.

As you can check, the chart has the numbers 1 to 31 many times and they are located in different places according to a trick.

This is the way to develop the game:

First, you ask someone who thinks a number from 1 to 31 and write secretly the number, so that you don't see it.

Then, you ask if the number is on the first column. In order to get the person read carefully all the numbers you would have to guide his reading pointing all the number from the top to the bottom.

After, you will do the same with the next column and so on until you finish all of them.

That is, there are five columns.

At this moment, you are able to guess the hidden number. How?

If the answer to the first question is positive, that is, the number is on the first column, you have to memorize 16 points, in the opposite way you do not add up anything.

If the answer to the second question is affirmative that means the number is on the second column and you have to add 8 points to the before account. In the opposite case you have to add nothing.

The punctuation for the third column is 4 points in a similar way; it is 2 points for the fourth column and 1 point for the fifth and last column.

That is all; the hidden number is the sum you get through the previous process.

You can colour each column in a different colour to improve the presentation and to identify better each of them.

25	31	21	26	13
29	15	5	10	7
22	9	12	6	3
19	11	15	14	15
16	14	7	3	1
31	10	14	15	5
26	13	6	11	9
17	12	13	7	11
20	8	30	27	31
27	24	31	22	29
30	28	23	30	27
21	25	4	31	25
18	30	20	18	23
23	26	29	19	21
28	27	22	2	19
24	29	28	23	17

## 1. NEGATIVE NUMBERS

Positive numbers are the numbers above zero: +5; +3; +7...

Negative numbers are the numbers below zero: -5; -3; -7...

We use this kind of numbers in temperatures, floors and bank accounts.

For instance,  $-7^{\circ}\text{C}$  means we are  $7^{\circ}$  below zero; -5 floor means 5 floor underground; €-345 means I owe €345.

To differentiate positives from negatives we use a + symbol for the positive numbers and a - for the negative numbers. This is the sign of the number.

Two numbers are opposite if the unique difference between them is the sign.

+5 and -5 are opposite numbers.

I gained 5 euros is +5 but I spent 5 euros is -5.

Absolute value of a number is the distance between the number and zero.

$|-7| = 7$ . If the temperature is  $-7^{\circ}\text{C}$  we are 7 degrees from zero.

Two opposite numbers have the same absolute value.  $|+6| = 6$ ;  $|-6| = 6$

## 2. INTEGERS

Integers are the natural numbers with the opposite numbers together.

This set of numbers is represented by the capital letter Z.

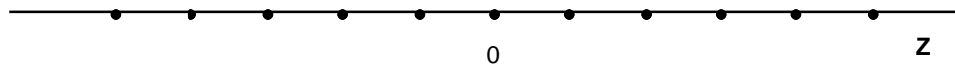
$Z = \{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4 \dots\}$

These numbers are infinite in two directions.

The set does not have an origin neither an end.

We put the negative numbers from zero to the left in the number line, the positive numbers from zero to the right.

Complete the next number line writing the numbers below the point.



## 3. OPERATIONS

We have the same operations than the natural numbers, that is, sum and multiplication and subtraction and division.

The rules to operate now are the followings:

1. To add a negative number is like to take away the number.

For instance,  $5 + (-3) = 5 - 3$

2. To subtract a negative number is like to add.

For instance,  $5 - (-3) = 5 + 3$

3. The product or division of two negative or positive numbers is positive.

4. The product or division of a negative number and a positive number is negative.

Example:

Complete the next chart according to the previous rules:

$+\cdot+=+$	$+5\cdot(+7)=$	$+\cdot-= -$	$+5\cdot(-6)=$
$-\cdot+= -$	$-9\cdot(+7)=$	$-\cdot-= +$	$-5\cdot(-7)=$

***Order of operations***

To do combine operations we have to follow the next order:

First. We do the brackets.

After. Multiplication and division

After. Addition and subtraction.

Operations with same hierarchy we do from left to right.

Example

$$(3 - 4) \cdot 5 - 2 \cdot (2 - 4 \cdot 3) + 5 =$$

**4. POWERS AND ROOTS OF INTEGERS****NUMBER OF SQUARE ROOTS OF AN INTEGER**

$\sqrt{0} = 0$  . It only has one square root.

$\sqrt{4} = \pm 2$  . It has two square roots.

$\sqrt{-9} = .$  It hasn't got any square root.

A negative number has no square roots.

There is not a number whose square can be  $-9$ .

## EXERCISES AND PROBLEMS

### 1. **NEGATIVE NUMBERS**

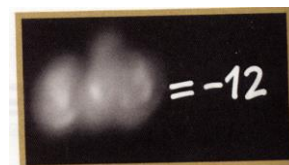
1. Put in order: 4; -5; 0; -2; -20; 12; 1
2. Draw the number line and write the numbers: 4; -1; 0; -4; 5; -2 and 6 on it.

### 2. **INTEGERS**

3. What letter represents the integers?  
What is the opposite of zero?  
What is the first integer number?  
Write the rules of signs in the multiplication.  
Write the rules of signs in the division.  
How many integers are there?  
What is the operation order?
4. Write mathematically the following sentences and work out the final temperatures:  
The temperature is eight degrees and it goes down fifteen degrees.  
Do the same in the following cases:
  - It is  $12^{\circ}$  and it goes down  $17^{\circ}$
  - It is  $8^{\circ}$  below zero and it goes up  $5^{\circ}$
  - It is  $25^{\circ}$  below zero and it goes up  $32^{\circ}$
  - It is  $15^{\circ}$  below zero and it goes down  $9^{\circ}$

### 3. **OPERATIONS**

5. Calculate:
  - a)  $(-2) \cdot (+6) =$
  - b)  $(-7) \cdot (-3) =$
  - c)  $(+2) \cdot (-5) =$
  - d)  $(+4) \cdot (+3) =$
  - e)  $(-1) \cdot (-8) =$
  - f)  $(+6) \cdot (-5) =$
6. Calculate:
  - a)  $(-2) \cdot (-7) \cdot (-1) =$
  - b)  $(+5) \cdot (-4) \cdot (-3) =$
7. Calculate:
  - a)  $(+28) : (+4) =$
  - b)  $(+35) : (-7) =$
  - c)  $(-21) : (-3) =$
  - d)  $(-8) : (-4) =$
  - e)  $(-3) : (-3) =$
  - f)  $30 : (-6) =$
8. The answer to the question on this blackboard is -12.  
Using multiplication and/or division signs, write down at least five different calculations that give this answer.
9. Using multiplication and/or division signs, write five



different calculations that give  $-18$  as result.

- 10.** Using multiplication and/or division signs, write five different calculations that give  $-20$  as result.

- 11.** Copy and complete the following multiplication tables.

**a**

$\times$	$-2$	$3$	$-4$	$5$
$-3$	6			
$6$				
$-2$				
$5$				

**b**

$\times$	$-1$	$-3$	$4$	
$-2$		6		
		12		
	$-5$			
$7$				$-42$

**c**

$\times$				$-8$
$-2$		$-12$		
	$-15$		$21$	
$4$			$28$	
		$-30$		

- 12.** Find the missing number in each calculation:

**a**  $2 \times -3 = \square$

**b**  $-2 \times \square = -8$

**c**  $3 \times \square = -9$

**d**  $\square \div -5 = -15$

**e**  $-4 \times -6 = \square$

**f**  $-3 \times \square = -24$

**g**  $-64 \div \square = 32$

**h**  $\square \times 6 = 36$

**i**  $-2 \times 3 = \square$

**j**  $\square \times -6 = -48$

**k**  $-2 \times \square \times 3 = 12$

**l**  $\square \div -4 = 2$

**m**  $5 \times 4 \div \square = -10$

**n**  $-5 \times \square \div -2 = -10$

**o**  $\square \times -4 \div -2 = 14$

- 13.** Work out the following:

a.  $-2 \times -2$ ; b.  $-4 \times -4$ ; c.  $(-3)^2$ ;  $(-6)^2$

- 14.** Work out the following operations:

a)  $-33 + 9 - 7 + 12$

b)  $-7 \cdot 4 \cdot (-3)$

c)  $-2 - 3 - 7$

d)  $-6 \cdot 4 \cdot 5$

- 15.** Take away the parentheses and calculate:

a)  $(-8) - (-4) + (-6) - (+2) - (-9) =$

b)  $(+7) - (+5) + (-11) - (-9) + (+4) =$

c)  $(+15) + (-13) - (+12) - (-10) =$

d)  $(-2) - (-8) + (-4) - (-6) - (+9) + (-7) =$

e)  $(+12) - (-14) - (+16) + (-18) - (-20) =$

- 16.** Work out the following:

a)  $-7 + 8$

b)  $-2 - 7$

c)  $+6 - 2 + 3$

d)  $-6 - 1 + 7$

e)  $-3 + 4 - 9$

f)  $-3 - 7$

g)  $-4 + (-6)$

h)  $+7 - (+6)$

i)  $-3 - 7 + (-8)$

j)  $-5 + -4 - (-7)$

- 17.** Work out:  $2 \cdot 6 + 10 - 5 \cdot 4 - 8 + 2 \cdot 7$

- 18.** Calculate step by step:

a)  $(3 - 4) \cdot 5 - 2 \cdot (2 - 4 \cdot 3) + 5 =$

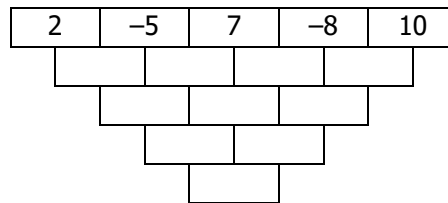
b)  $-7 + 5 \cdot (-3) + 2 =$

c)  $-5 \cdot (2 - 4 \cdot 3) + 15 =$

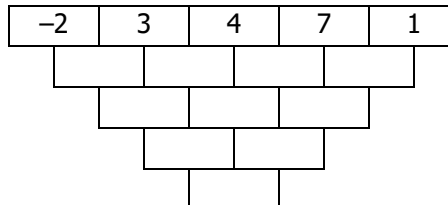
- 19.** Calculate step by step:  $5 - 2 \cdot (3 + 2) - 4 \cdot (4 - 7) =$

- 20.** Work out:  $20 : (6 - 8) - (4 - 2) + 6 \cdot 5 : 3 =$

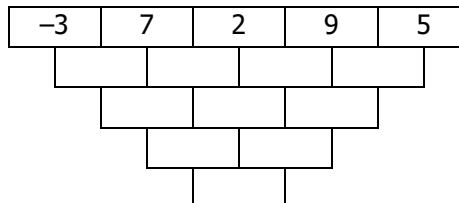
- 21.** Do the following operations:  
 a)  $4 \cdot (10 - 2) - 2 \cdot (-3 + 15) + 9$   
 b)  $8 - [7 - (-2 + 5) - 1] + 4$   
 c)  $5 \cdot (8 - 2 + 3) - (-4) \cdot [6 - (2 + 7)]$
- 22.** Work out:  $5 - (-3 + 4 - 2) - 3 \cdot (2 + 5 - 4) =$
- 23.** Calculate step by step:  $8 - 15 \cdot 5 - 64 : 8 + 4 \cdot 8 : 2 =$
- 24.** Calculate step by step:  $-3 \cdot (-5) - 4 \cdot (2 - 7 \cdot 2) + 14 : 2$
- 25.** Calculate step by step:  $2 \cdot [-3 + (4 - 5 \cdot 2)] - 7 \cdot (-5) \cdot 3 - 6 : 3 =$
- 26.** Calculate step by step:  $-7 \cdot (2 - 5 \cdot 4) - (-2 \cdot 5 + 7 \cdot 3) =$
- 27.** Work out:  $3 \cdot (-5 - 2 \cdot 8) - [2 \cdot (-3) - 7 \cdot (-2)] =$
- 28.** Do the following operations:  
 a)  $-3(2 - 7) + 5 \cdot (-6) : 3 =$   
 b)  $16 : (4 - 12) - 3(25 - 30) =$
- 29.** The temperature in a room is  $25^{\circ}\text{C}$  in winter. We stop the heating and thus it goes down  $5^{\circ}\text{C}$  per day. What's the temperature 6 days later?
- 30.** In this 'wall', subtract the right-hand from the left-hand number to find the number in the brick below.



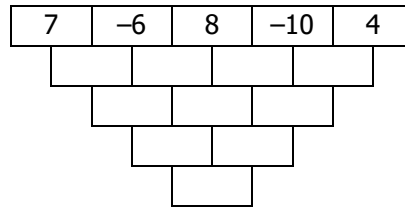
- 31.** In this 'wall', add the right-hand from the left-hand number to find the number in the brick below.



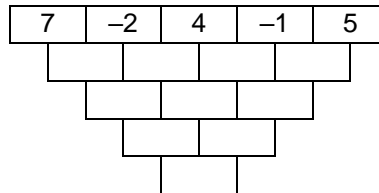
- 32.** In this 'wall', subtract the right-hand from the left-hand number to find the number in the brick below.



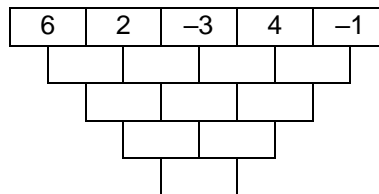
- 33.** In this wall add the right-hand from the left-hand to find the number in the brick below.



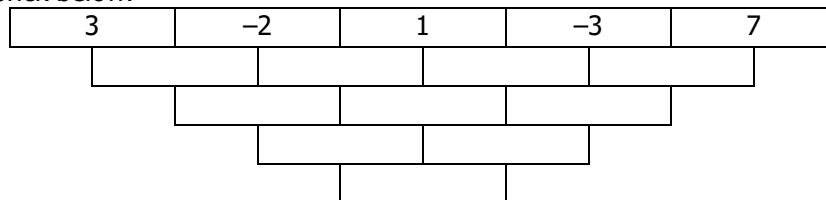
- 34.** In this 'wall', subtract the right-hand from the left-hand number to find the number in the brick below.



- 35.** In this 'wall', add the right-hand from the left-hand number to find the number in the brick below.



- 36.** For this 'wall', subtract the right-hand from the left-hand number to find the number in the brick below.



- 37.** Find the missing number in each calculation:

a)  $-7 + \quad = -13$    b)  $-8 + \quad = 15$    c)  $\quad + (-2) = -5$    d)  $\quad + (-6) = 9$   
 e)  $7 - \quad = -7$    f)  $-3 - \quad = -3$    g)  $\quad - (-2) = -8$    h)  $5 - \quad = 10$

- 38.** Find the missing number in each calculation:

a)  $-5 + \quad = -11$    b)  $-3 + \quad = 12$    c)  $\quad + (-2) = 5$    d)  $\quad + (-6) = -9$   
 e)  $7 - \quad = -7$    f)  $-3 - \quad = -3$    g)  $\quad - (-4) = -8$    h)  $2 - \quad = 10$

- 39.** Complete the following operations putting the missing numbers:

a)  $-5 + \quad = -12$    b)  $-6 + \quad = 15$    c)  $\quad + (-7) = -5$    d)  $\quad + (-4) = 9$   
 e)  $9 - \quad = -9$    f)  $-7 - \quad = -7$    g)  $\quad - (-8) = -5$    h)  $3 - \quad = 9$

- 40.** Find the missing number in each calculation:

a)  $-4 + \quad = -12$    b)  $-2 + \quad = 15$    c)  $\quad + (-3) = -5$    d)  $\quad + (-5) = 9$   
 e)  $3 - \quad = -3$    f)  $-1 - \quad = -1$    g)  $\quad - (-4) = -5$    h)  $7 - \quad = 9$

#### 4. Powers and roots of integers

- 41.** Calculate: a)  $(-2)^6$ ; b)  $(-1)^5$ ; c)  $(-3)^3$ ;  $(-7)^0$



- 42.** Simplify the following expressions writing as a unique power using the properties of powers:

a)  $(-1)^4 \cdot (-1)^2 \cdot (-1)^5$ ; b)  $(-2)^4 \cdot (-2)^3 \cdot (-2)^5$